TURBULENT FLOW OF DILUTE EMULSIONS

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An equation is obtained to determine the coefficient of hydraulic drag of dilute emulsions by using the turbulent viscosity concept and the phenomenon of quenching turbulent pulsations. The results of the theory are compared with experiment.

If some mass of fluid is incident in a turbulent stream which does not mix with the fluid and has a sufficiently high degree of turbulence, then fractionation of this fluid under the effect of the turbulent pulsations occurs [1-3]. In this case a dilute emulsion is formed for a low content of the dispersing fluid. The least diameter of the droplets of such an emulsion will exceed the internal scale of the turbulent pulsations and can be determined as a function of the intraphasal tension σ, the density of the dispersion medium ρ_1, the inner diameter D of the pipe, and the mean flow velocity w by means of the Kolmogorov formula [2, 3]

$$d = 2\sqrt{2}\left(\frac{\sigma}{k\rho_1}\right)^{3/5} \frac{D^{2/5}}{w^{6/5}}, \tag{1}$$

where $k \approx 0.5$ is the drag coefficient for flow around a drop.

The viscosity conception developed in the Millionshchikov semiempirical theory of turbulence [4-6] is used to describe the turbulent flow of dilute emulsions; hence the influence of the dispersed fluid on the coefficients of dynamic μ_e and turbulent μ_{te} viscosity of the emulsion is taken into account.

Dilute emulsions behave similarly to simple fluids and are subject to the Newton and Poiseuille laws. Let us write the equation of motion of dilute emulsions in a pipe as

$$(\mu_e + \mu_{te})\frac{du}{dy} = \tau, \tag{2}$$

where u and τ are the velocity and tangential stress at a distance y from the pipe wall. Under the assumption of axial symmetry, the tangential stress at the given section is related to the tangential stress at the wall τ_ω by

$$\tau = \tau_\omega(1 - y_0),$$

where y_0 is the dimensionless distance from the wall defined by the ratio between y and the pipe radius r, i.e., $y_0 = y/r$.

The influence of dispersed fluid globules on the coefficient of dynamic viscosity of a dilute emulsion is manifest in the fact that the dynamic viscosity of the emulsion μ_e increases with the growth in the content of the dispersion phase β which exceeds the viscosity of the dispersion medium μ_1. Brinkman [7] obtained

$$\mu_e = \mu_1(1 - \beta)^{-2.5} \tag{3}$$

for the case when the drops move independently.

Groznyi. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 116-120, May-June, 1975. Original article submitted August 6, 1974.

TABLE 1. Physical Properties of Fluids at a Temperature of 20°C

Working fluid	Kinematic viscosity, m²/sec	Density, kg/m³	Interphasal tension on the transformer oil—water interface, N/m
Water	$1{,}7 \cdot 10^{-6}$	998	$44{,}8 \cdot 10^{-3}$
Transformer oil *	$24{,}3 \cdot 10^{-6}$	896	

*The surfactants in transformer oil are asphalt-resins.

For dilute emulsions, (3) yields good agreement with test data and can be used in the motion equation (2).

For flows of emulsions in a pipe the scale for the pulsation velocities in the region of developed turbulence is the dynamic velocity corresponding to the tangential stress at a given radius [4-6]:

$$v_{*y\,e}=v_{*e}\sqrt{1-y_0}, \tag{4}$$

where $v_{*e} = \sqrt{\tau\omega/\rho_e}$ is the dynamic velocity. The density of the emulsion ρ_e is hence determined additively $\rho_e = \rho_1 (1-\beta) + \rho_2\beta$, where ρ_2 is the density of the disperse phase.

Equation (2) can be represented as

$$(\nu + \nu_{te})\frac{du}{dy}=v_{*e}^2(1-y_0)=v_{*ye}^2, \tag{5}$$

where ν_e is the coefficient of kinematic viscosity of the emulsion ($\nu_e = \mu_e/\rho_e$); ν_{te} is the turbulent kinematic viscosity ($\nu_{te} = \mu_{te}/\rho_e$).

Let us define the turbulent kinematic viscosity as the product of the dynamic velocity v_{*ye} at a given radius and the mixing path l_e.

$$\nu_{te}=v_{*ye}l_e. \tag{6}$$

Then taking account of (4) and (6), Eq. (5) becomes

$$\left(\nu_e+v_{*e}l_e\sqrt{1-y_0}\right)\frac{du}{dy}=v_{*e}^2(1-y_0).$$

It is seen from this equation that for sufficiently high values of the Reynolds number when the viscosity can be neglected, a change in l_e along the radius in conformity with the formula

$$l_e=a_e(y-\delta_\Lambda)\sqrt{1-y_0},$$

corresponds to the logarithmic profile, where a_e is a dimensionless coefficient and δ_Λ is the thickness of the laminar sublayer. For single-phase fluids the value of the dimensionless coefficient and the laminar sublayer thickness are determined in [4-6] on the basis of processing experimental data: $a = 0$ for laminar flow, $a = 0.39$ for developed turbulent flow, and $\delta = v_*\delta_\Lambda/\nu = 7.8$.

The influence of globules of dispersed fluid on the turbulent kinematic viscosity is manifest in the diminution of the mixing pathlength as compared with the turbulent flow of a single-phase fluid.

Since a film of surfactants adsorbed on the fluid interface hinders penetration of the pulsating motions within the globules, then the diminution of the mixing pathlength during turbulent flow of an emulsion is determined, first of all, by a diminution in the volume in which turbulent energy dissipation occurs (it is necessary to eliminate the volume occupied by the globules of dispersed phase from the total volume of the turbulent stream core), i.e., the factor $(1-\beta)$ should be introduced in the dimensionless coefficient of the mixing pathlength. Moreover, since the dimension of droplets of the emulsion under consideration exceeds the internal scale of turbulence of the dispersion medium λ_0, some quenching of the turbulent pulsations occurs on the surface of these droplets. If it is considered that complete quenching of the turbulent pulsations occurs in the dispersion medium in an emulsion with close packing of globules of diameter d_p, such that the clearance between them does not exceed λ_0, then the efficiency of quenching the turbulent pulsations on the surface of dilute emulsion globules can be taken into account by the factor $(1 - S/S_p)$ to the dimensionless coefficient of the mixing pathlength, where S/S_p is the ratio between the interphasal surfaces of the dilute and most compact emulstions. Therefore

$$a_e = 0.39(1-\beta)(1 - S/S_p).$$

The fraction of dispersed phase by volume for the closest arrangement of globules in the compact emulsion is $\beta_p = .741$ [8].

Hence

$$\frac{S}{S_p} = \frac{\beta}{0.741}\frac{d_p}{d}. \tag{7}$$

Examining the model of the closest globule arrangement, we find from geometric representations that the greatest dimension of the clearance between globules is $h = 0.365 d_p$. The internal scale of turbulence of the dispersion medium λ_0 is determined from the condition that the Reynolds number for motion of the scale λ_0 is one [3]:

$$\lambda_0 = \frac{D}{\mathrm{Re}_1^{3/4}} = \left(\frac{D^{1/3}\mu_1}{w\rho_1}\right)^{3/4}.$$

Equating h and λ_0 we find that

$$d_p = 2.74\left(\frac{D^{1/3}\mu_1}{w\rho_1}\right)^{3/4}. \tag{8}$$

Substituting (1) and (8) into (7), we obtain

$$S/S_p = 0.863\beta M^{0.15},$$

where $M = \mu_1^5 w^3 / D\rho_1\sigma^4$ is a dimensionless parameter.

Consequently, we finally have for the dimensionless coefficient of the mixing pathlength

$$a_e = 0.39(1-\beta)(1 - 0.863\beta M^{0.15}). \tag{9}$$

The laminar sublayer thickness can evidently be considered analogously to a single-phase fluid $\delta = v_{*e}\delta_\Lambda/\nu_e = 7.8$.

Therefore, the equation of motion of a dilute emulsion in a pipe becomes

$$[b_e + a_e(y_0 - \delta_0)(1 - y_0)]\frac{du/v_{*e}}{dy_0} = 1 - y_0,$$

where

$$b_e = \nu_e / r v_{*e},\ \delta_0 = \delta_\Lambda / r,\ a_e = 0 \quad \text{for } y_0 < \delta_0;$$

$$a_e = 0.39(1-\beta)(1 - 0.863\beta M^{0.15}) \quad \text{for } y_0 \geqslant \delta_0.$$

Integrating this equation taking account of a smooth merger with the laminar sublayer, we have

$$\frac{u}{v_{*e}} = \frac{1}{b_e}\left(y_0 - \frac{y_0^2}{2}\right) \quad \text{for } y_0 < \delta_0, \tag{10}$$

$$\frac{u}{v_{*e}} = \frac{1}{2a_e}\left\{\ln\left[1 + \frac{a_e}{b_e}(y_0 - \delta_0)(1 - y_0)\right] + \frac{1-\delta_0}{\sqrt{\Delta}}\ln\frac{\{\sqrt{\Delta} + [2y_0 - (1+\delta_0)]\}[\sqrt{\Delta} - (\delta_0 - 1)]}{\{\sqrt{\Delta} - [2y_0 - (1+\delta_0)]\}[\sqrt{\Delta} + (\delta_0 - 1)]}\right\} + \delta\left(1 - \frac{\delta_0}{2}\right)$$

$$\text{for } y_0 \geqslant \delta_0, \text{ where } \sqrt{\Delta} = \sqrt{4\frac{b_e}{a_e} + (1 - \delta_0)^2}.$$

The formulas (10) yield the velocity distribution over the whole range of variation of y_0 between 0 and 1, where the condition $du/dy = 0$ is satisfied on the pipe axis, i.e., for $y_0 = 1$.

The drag coefficient for an emulsion flowing in pipes is defined by the formula

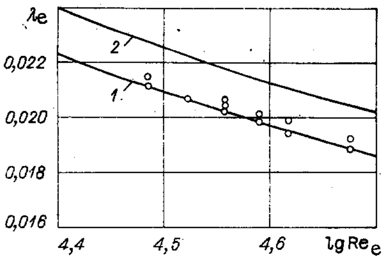

Fig. 1

$$\lambda_e = 2\frac{D}{L}\frac{\Delta p}{\rho_e w^2} = 8\left(\frac{v_{*e}}{w}\right)^2,$$

where Δp is the pressure drop in a length L.

The mean value of the velocity is determined by the integral

$$w = 2\int_0^1 (1 - y_0)\, u\, dy_0.$$

Integration is hence separated into two sections, from 0 to δ_0, where $a_e = 0$ and from δ_0 to 1, where a_e is defined by means of (9). Following [4-6], in the integration between δ_0 and 1 we use the replacement of (10) for this interval by the expression

$$\frac{u}{v_{*e}} = \frac{1}{a_e}\ln\left[1 + \frac{a_e}{b_e}(y_0 - \delta_0)\right] + \delta + f(y_0/b_e,$$

which differs from (10) for small values of b_e by just the small correction $f(y_0/b_e)$ which appears just near the pipe axis.

The formulas for w/v_{*e} and λ_e are

$$\frac{w}{v_{*e}} = \frac{1}{2}\operatorname{Re}_e b_e = \frac{1}{b_e}\left(\delta_0^2 - \delta_0^3 + \frac{\delta_0^4}{4}\right) + \frac{b_e^2}{a_e^3}\left[\alpha^2\left(\ln\alpha - \frac{3}{2}\right) + 2\alpha - \frac{1}{2}\right] + \delta - \varepsilon; \qquad (11)$$

$$\lambda_e = \frac{8}{\left(\frac{\operatorname{Re}_e b_e}{2}\right)^2},$$

where

$$\operatorname{Re}_e = \frac{wD\rho_e}{\mu_e}, \quad \alpha = 1 + \frac{a_e}{b_e}(1 - \delta_0),$$

ε is a small correction related to the function $f(y_0/b_e)$ which can be neglected in practice. Formulas (11) yield a parametric dependence between Re_e and λ_e, where b_e is the parameter.

Experimental investigations to determine the hydraulic drag coefficient λ_e were conducted on an apparatus described in [9] for the flow of a dilute transformer oil emulsion in water in a 39.4-mm-diameter pipe at a temperature of 19 ± 1°C.

The fluids whose physical properties are presented in the table were delivered to the experimental section by extrusion by air from reservoirs so that the formation of the emulsion in the pipeline occurred only under the effect of turbulent pulsations.

Given in Fig. 1 is a comparison between the results of experiments and computations using (11) for an emulsion with a $\beta = 0.1$ dispersed phase content. Curve 2 shows the drag law for a pure fluid. It is seen that the drag coefficient for a dilute emulsion (curve 1) is substantially below that for the pure fluid.

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